Simulating Income Distributions (2)

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Gini Coefficients in Variable Economies

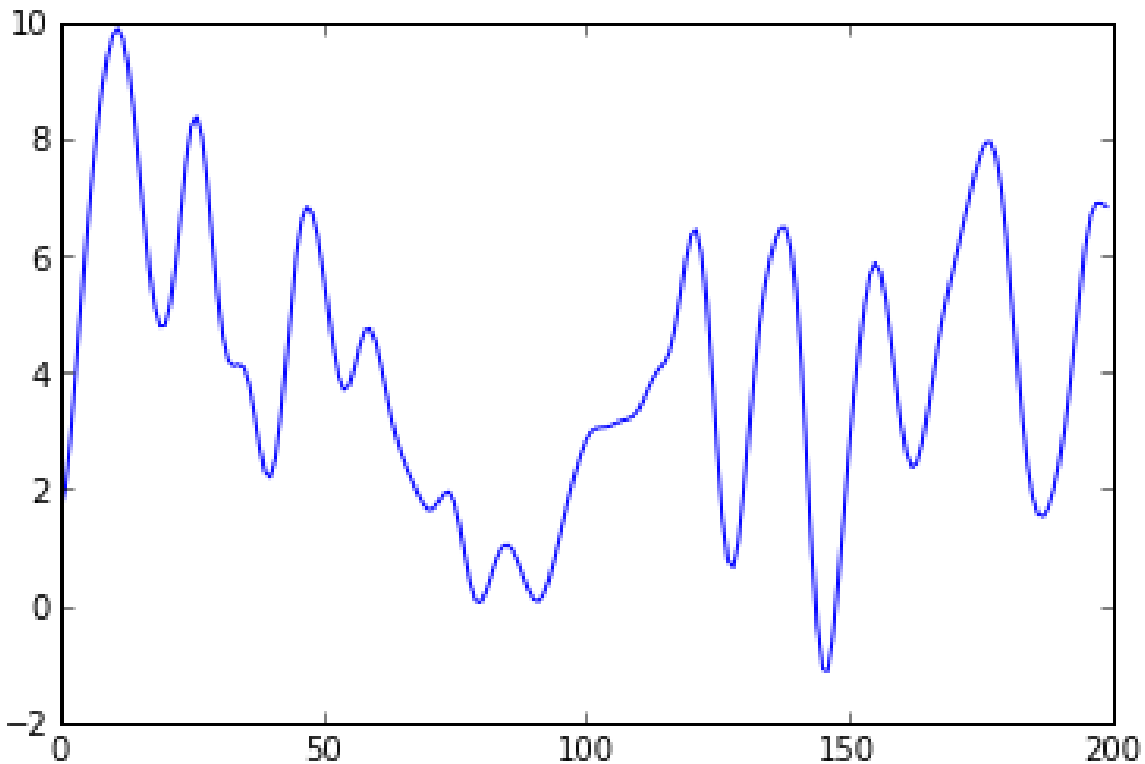
HYPOTHESIS: The variability in economic conditions over time (boom/bust cycles) together with different times when people are born increases the Gini coefficient of the income distribution of a society.

An additional source of variation could be that growth rates are themselves variable over time, as the economy undergoes busts and booms.

```
from scipy.ndimage import filters

def ecycles(target=4.0,sd=8.0,smooth=3.0,n=100):
    rates = randn(n)*sd+target
    rates = filters.gaussian_filter(rates,smooth)
    rates += target-mean(rates)
    return rates
```

```
1 rates = ecycles(n=200)
2 plot(rates)
3 print mean(rates)
```

```
4.0
```



For the simulation, things now get trickier, since we actually need to keep track of populations over time, aging, etc. This is actually turning into *agent based modeling*. Each agent has a number of variables:

- a lifetime
- an age
- a current income

In an OOL, we might actually make each individual an object, but in an array language, we can also write this naturally using array.

Furthermore, we start the simulation up with everbody at the same age, but then let the population equillibrate over time.

```
def random_lifetimes(N=N):
    return (empsample(adist,n=N)+rand(N))*5.0
```

```
def empsample(dist,n=1):
    """Given an array representing a histogram, return a random bin
        number
    according to that histogram."""
    cdist = add.accumulate(dist)
    cdist = cdist*1.0/amax(cdist)
    result = [bisect(cdist,rand()) for i in range(n)]
    if n==1: return result[0]
    else: return array(result,'f')
```

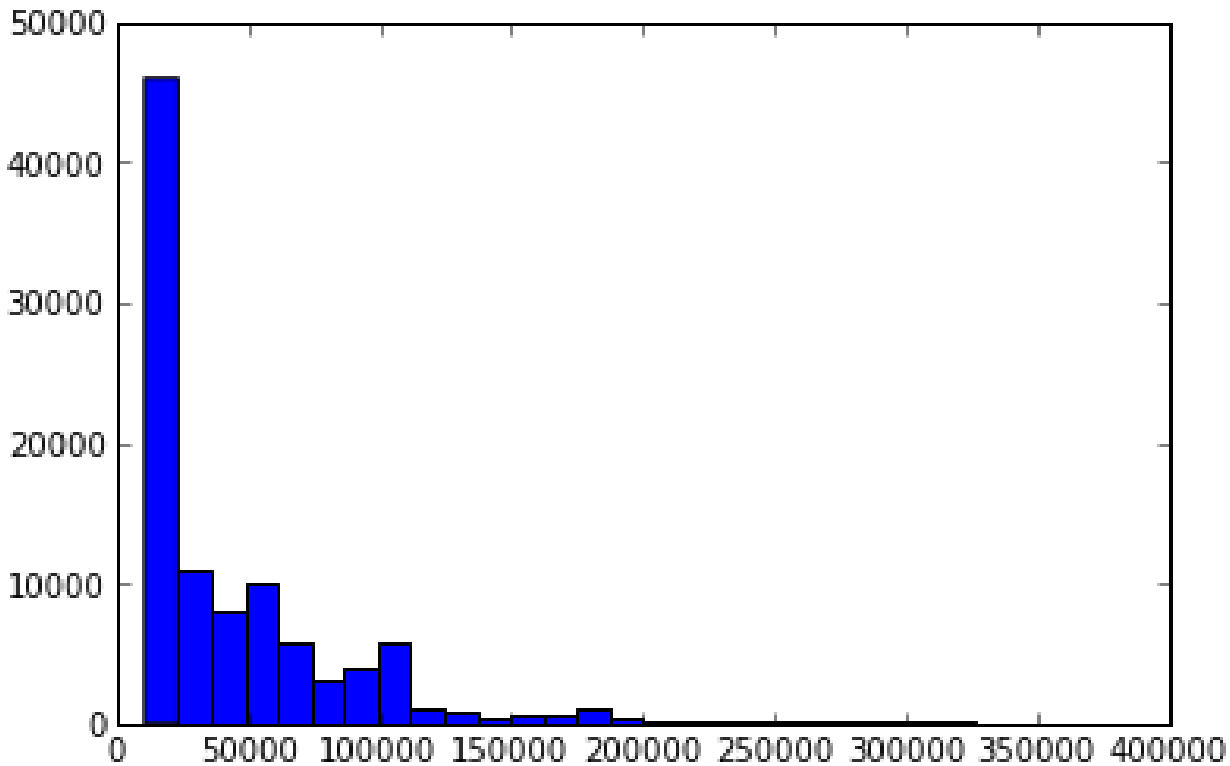
```
starting_income = 10000.0
lifetimes = random_lifetimes()
ages = zeros(N)
incomes = starting_income*ones(N)
```

```
for y,g in enumerate(rates):
    die = find(ages>lifetimes)
    ages[die] = 0
    lifetimes[die] = random_lifetimes(len(die))
    incomes[die] = starting_income
    incomes *= 1.0+g/100.0
    ages += 1
```

```
1 incomes
```

```
array([ 270688.08170829,   17288.27032543,   46999.59975542, ...,
         76203.0166551 ,   17288.27032543,   34790.09095561])
```

```
1 _=hist(incomes,bins=30)
2 print gini(incomes)
```

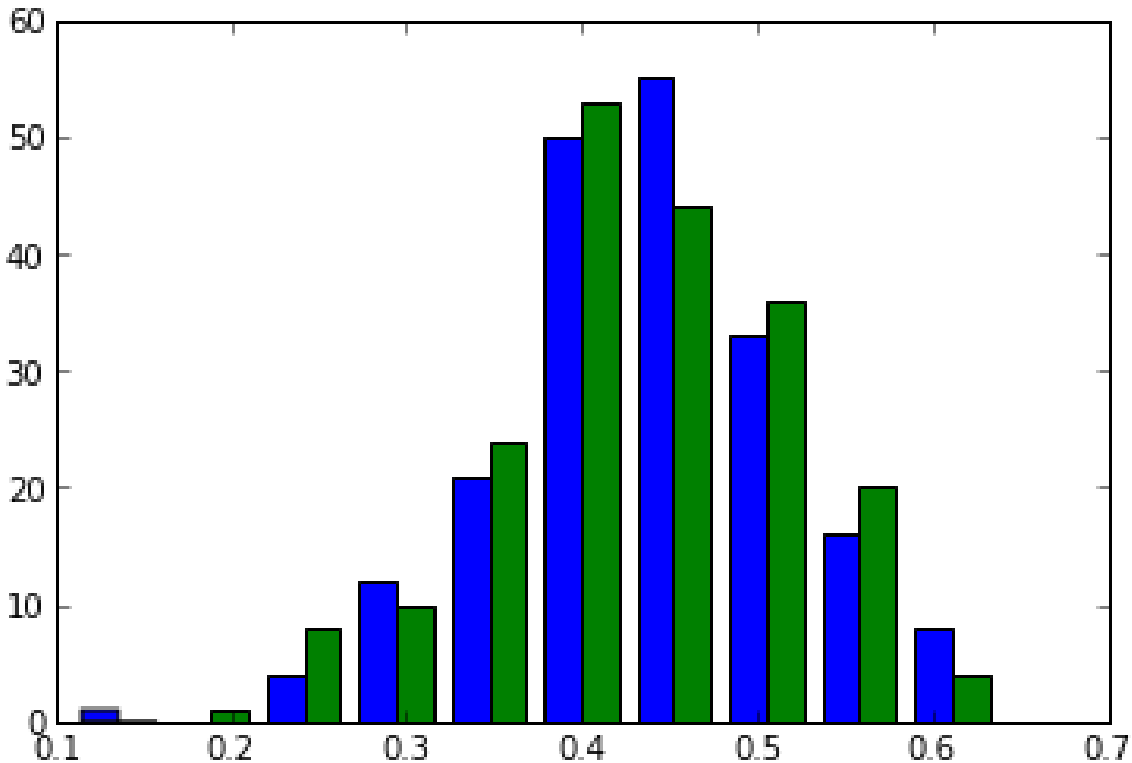


```
def experiment(target=2.0,smooth=3.0,sd=8.0,n=200):
    rates = ecycles(target=target,smooth=smooth,sd=sd,n=n)
    starting_income = 10000.0
    lifetimes = random_lifetimes()
    ages = zeros(N)
    incomes = starting_income*ones(N)
    for y,g in enumerate(rates):
        die = find(ages>lifetimes)
        ages[die] = 0
        lifetimes[die] = random_lifetimes(len(die))
        incomes[die] = starting_income
        incomes *= 1.0+g/100.0
        ages += 1
    return gini(incomes)
```

```
ginihv = []
for i in range(200):
    ginihv += [experiment(target=4.0,sd=8.0)]
    print i,ginihv[-1]

ginilv = []
for i in range(200):
    ginilv += [experiment(target=4.0,sd=8.0)]
    print i,ginilv[-1]
```

```
1 _=hist([ginilv,ginihv])
2 print mean(ginilv),mean(ginihv)
```

```
0.434198721774 0.433194689808
```



Observations:

- The hypothesis is false.
- Gini indexes have high variance in both cases
- this means that differences in Gini index for countries with different economic histories may not be meaningful
- there is (surprisingly) no significant difference in average Gini index

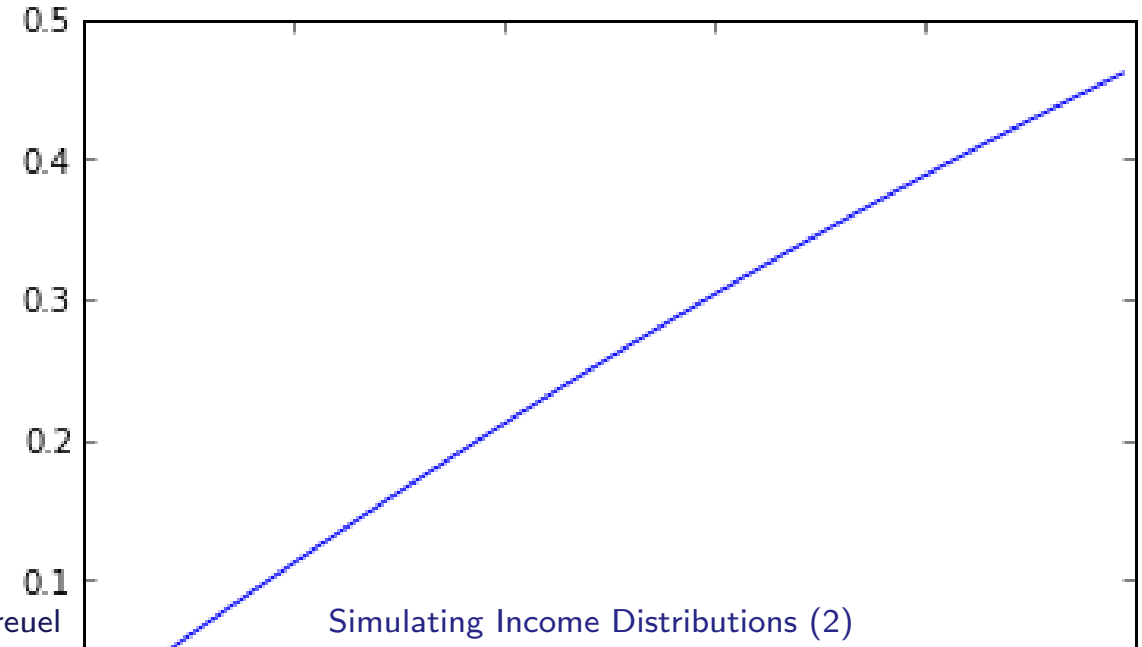
Competitive Economies

HYPOTHESIS: Competition together with slight differences in skill can produce large inequality.

Let's look at a completely different economic model.

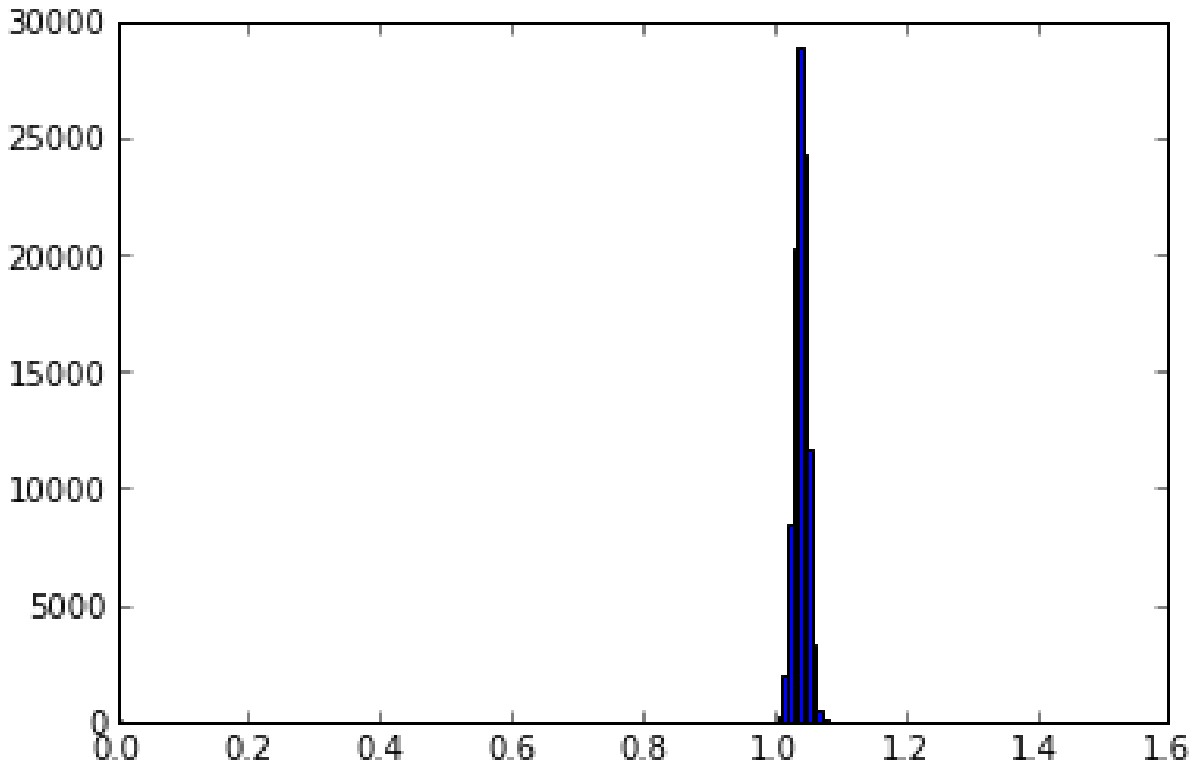
- Here, everybody starts out with the same starting capital (USD 10000).
- People have slightly different skills at multiplying that capital, a normal distribution with a small spread.
- They now exist in a competitive environment, where the return on investment is proportional to their skill.
- Overall growth of the economy is limited to an annual 4
- Note that this limit is usually achieved through a combination of inflation and other mechanisms.

```
wealth = 10000.0 * ones(N)
global_growth = 0.04
skill = maximum(1.01,1.04+0.01*randn(N))

gs = []
for i in range(100):
    rgrowth = (1.0-skill)*wealth
    growth = global_growth*sum(wealth)*rgrowth/sum(rgrowth)
    wealth += growth
    gs.append(gini(wealth))

plot(gs)
```



Here we see that slight differences in starting conditions (skills) produce great differences in wealth over the long term.

```
1 _=hist(skill,bins=200,range=(0,1.5))
```



Fairness

- Is the resulting inequality "fair"?
- Regardless of whether it is fair or not, very high inequality may cause problems.
- What would be "fair" solutions to limiting inequality?

Question

- Can you design tax or redistribution policies that “fix” this?
  - Can you do so fairly?